

## Math 107, Spring 2015: Midterm II Study Guide

You should be comfortable with all the examples from class and the homework questions you had to do on all homework assignments, the practice questions for both midterms, and the actual midterms. In addition to these, I will give you some practice problems on the new material we have covered after the second midterm on Friday in class.

If by any chance you have not done the homework by yourself, I again advise you to redo it all by yourself now.

Note that we have covered things in a different order than the book, we did not cover all the material in the book, and we covered a few extra things and talked about a few things slightly differently than the book. You are not responsible for anything we haven't covered in class or on the homework. However, you are responsible for everything we have done in class and on the homework, so please do review your notes and homework assignments.

If you have any questions or concerns, please email me or come to see me on Monday during my review office hours.

### Syllabus

Here are some things you should be comfortable with. I am mostly copying and pasting the previous 2 study guides (with some slight edits) and adding the new topics to them.

- integrals of rational functions
  - what a rational function is;
  - how to recognize an improper rational function and use long division to write it as the sum of a polynomial and a proper rational function;
  - how to write down the partial fraction decomposition of a proper rational function (make sure you really understand how this works in general);
  - how to integrate rational functions (for this you have to be comfortable with integration techniques, so all the practice from the examples from class and homework is crucial; I suggest you review from class and homework how to do the integrals of the functions that show up a lot such as  $\frac{1}{x}$  and  $\frac{1}{1+x^2}$ )
- improper integrals
  - what kinds of improper integrals there are;
  - the limit definitions of all of them (you have understand which limit you have to take for each kind of improper integral, so trying to understand the definitions first is a very good idea)

- which improper integrals are defined as a sum of limits (and know that these are convergent only if both limits in the sum exist)
  - how to compute improper integrals straight from the limit definition
  - when you can apply the comparison test and be able to use it to decide whether an improper integral is convergent or divergent.
- probability:
    - the multiplication principle;
    - what permutations and combinations are and the difference between;
    - what a sample space and an event is and how to set these up in examples;
    - what properties a probability function satisfies, including the 3 propositions we have proved in class (we will see examples this week of how they come in in practice to make probability computations easier);
    - how to calculate the probability of an event when all the outcomes in the sample space are equally likely;
    - what conditional probability is;
    - what independent events are and how to check if two events are independent;
    - how to compute the probably of two events both happening;
    - Bayes’s formula - how to set up events, how to set up the formula , and how to use it to find conditional probabilities.
- discrete random variables
    - what discrete random variable are;
    - what the probability mass function is;
    - what the cumulative distribution function is (and how to plot it);
    - how to find the cumulative distribution function from the mass function and vice-versa;
    - Bernoulli and binomial random variables;
    - definition of mean  $E(X)$  and variance  $var(X)$  of a random variable  $X$ ;
    - finding  $E(g(X))$  for a real-valued function  $g$ ;
    - using the propositions we proved about  $E(X)$  and  $var(X)$  to compute these in specific examples;
- continuous random variables
    - what a continuous random variable is;
    - what a probability density function is;

- what it means for  $f$  to be the probability density function for a continuous random variable  $X$  (i.e. how the probabilities that the values on  $X$  lie in certain intervals are calculated in terms of  $f$ );
- what the cumulative distribution function is;
- how the probability mass function and the cumulative distribution function are related and how to compute one in terms of the other;
- expected value  $E(X)$ ,  $var(X)$  and the properties of these that we have shown;

NOTE: you should be comfortable with integration, improper integrals and L'Hopital – as you have seen these show up in computations of  $E(X)$  and  $var(X)$  of continuous random variables.

- differential equations

- what a differential equation is;
- how to check if a particular function is a solution of a differential equation;
- separable differential equations: when is an equation separable, general method of solution, finding a specific solution given an initial condition;
- general solution of  $\frac{dy}{dt} = f(t)g(y)$ , and finding a specific solution using an initial condition
- understanding how to find the constant solutions (these are not the constants that show up in the general solution, but the constant functions which are solutions to the differential equation);

NOTE: you should be comfortable with integration by parts – as you have seen, this shows up in finding the solutions to such equations.

- linear algebra

- addition, subtraction, scalar multiplication of matrices;
- matrix multiplication for matrices (of any size);
- identity matrix, inverse matrices (definition and formula for inverse of a 2x2 matrix)
- determinant of a 2x2 matrix, invertible/non-invertible matrices;
- using matrices to solve linear equations, when is there a unique solution (and when are there 0 or infinitely many solutions);
- linear transformations  $\mathbb{R} \rightarrow \mathbb{R}$  and their geometric interpretations (in the cases where we can give an obvious interpretation)
- eigenvectors and eigenvalues of a 2x2 matrix (definition and what the method of finding them is)

- systems of differential equations
  - what a linear system of differential equations is;
  - how to check if a vector consisting of two functions is a solution to a system of differential equations;
  - what the general solution to a system of differential equations is (and what ingredients we need to describe it - this will require knowledge of the linear algebra section);
  - what an initial value constraint for a system of differential equations consists of;
  - how to find the specific solution to a system of differential equations for an initial value constraint;
- multivariable calculus
  - what 2-variable functions are (i.e., functions  $f : D \rightarrow \mathbb{R}$ , where  $D \subseteq \mathbb{R}^2$ );
  - finding the largest possible domain and what the range of a function of 2 variables;
  - how to plot the level curves for a 2-variable function, and how to think of the graph in terms of these;
  - what a limit of a 2-variable function at a point means and how a limit can fail to exist;
  - how to show that a limit exists and how to show that a limit does not exist;
  - what continuity means for functions of 2 variables.

NOTE: I won't include anything regarding partial derivatives since we have glossed over them too quickly. For fun, think about them more and also think about what a double integral over a domain of integration that ranges over two variables might measure. (I said for fun, obviously this won't be on the exam.)